Applied Probability Workshop

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Book of Abstracts

Abstracts of the talks

Gibbs point process with non summable pairwise interaction

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In this talk, we discuss the question of Gibbs point processes in $\mathbb{R}^d$ with pairwise interactions that are not integrable at infinity. A standard example is the Riesz potential of the form $\varphi(x) = \frac{1}{|x|^s}$, where $s < d$. This setting has a long history, notably because the case $s = d - 2$ corresponds to the classical Coulomb potential, which arises from electrostatic theory. We will first address the existence of the process in the infinite volume regime when a neutralizing background is introduced (this model is known as Jellium in theoretical physics). Subsequently, we will discuss the rigidity of such point processes, specifically hyper-uniformity and number rigidity. We will provide a state-of-the-art review and present numerous conjectures and open problems.

On the topology of higher-order age-dependent random connection models

Christian P. Hirsch
Aarhus Univ.

Preferential attachment is a popular mechanism for generating scale-free networks. While it offers a compelling narrative, the underlying reinforced processes make it difficult to rigorously establish subtle properties. Recently, age-dependent random connection models were proposed as an alternative that is capable of generating similar networks with a mechanism that is amenable to a more refined analysis. In this talk, we analyze the asymptotic behavior of higher-order topological characteristics such as higher-order degree distributions and Betti numbers in large domains. We demonstrate the practical application of the theoretical results to real-world datasets by analyzing scientific collaboration networks based on data from arXiv. This talk is based on joint work with Péter Juhász.

The sufficient digits of continuous random variables

Jesper Møller
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A general setting for nested subdivisions of a bounded real set into intervals defining the digits $X_1, X_2, ...$ of a random variable X with a probability density function f is considered. Under the weak condition that f is almost everywhere lower semi-continuous, a coupling between X and a non-negative integer-valued random variable N is established so that $X_1, ..., X_N$ have an interpretation as the "sufficient digits", since the distribution of $(X_{N+1}, X_{N+2}, ...)$ conditioned on $(X_1, ..., X_N)$ does not depend on f. The importance of this coupling result and some suggestions and open problems for future research are discussed. Related papers are available on arXiv: `arxiv.org/abs/2404.09525`, `arxiv.org/abs/2307.06685`.

Using global quantile regression in stochastic geometry

Tomáš Mrkvička
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Most of the object in stochastic geometry are usually characterized by one or several summary functions. Thus when analyzing linear model where the response variable is an object from stochastic geometry, e.g. point pattern or random closed set, the functional methods can be used. The crucial problem of this idea is that the summary function of the stochastic geometry object has variability, that is dependent on the size of the object. Since different objects have different sizes, this variability should be treated in the model, but it is usually possible only for the simplest models as Poisson point pattern. We are coming with completely different solution of this problem through nearest neighbor distances. These characteristics can be calculated for every point of the point pattern and thus the linear model with point pattern as the response variable can be replaced by quantile regression, where each distance represent one observed response in the quantile regression. Recently we developed a global quantile regression method that can test significance of a certain covariate when all quantiles of the response distribution are taken into account, i.e. the whole distribution in one test. Thus such a model can answer question if a certain spatial covariate influences the structure of the point pattern or if a certain nonspatial covariate influences the structure of the replicated point patterns. The simplest model can be ANOVA models with point pattern as the response variable. The global quantile regression is based on pointwise coefficients, permutations and global envelope tests. The global envelope test serves as the multiple test adjustment procedure under the control of the family-wise error rate and provides the graphical interpretation which automatically shows the quantiles or the levels of categorical covariate responsible for the rejection. The Freedman-Lane permutation strategy showed liberality of the test for extreme quantiles, therefore we propose four alternatives that work well even for extreme quantiles and are suitable in different conditions. The shortcoming of this approach is the spatial dependence of the

response data, but this can be solved by using pointwise spatial quantile regression that is already developed and the rest can be done as in global quantile regression. This talk will present this new direction of research in stochastic geometry together with the already developed global quantile regression.

Some notes about Poisson–Laguerre tessellation with unbounded weights

Martina Petráková
Charles Univ.

The object of our research is the Poisson–Laguerre tessellation, i. e. a random Laguerre tessellation whose generator is a Poisson marked point process with intensity t. We are interested in the asymptotic behaviour (as $t \to \infty$) of functionals of the tessellation – e. g. the perimeter of the cells, the ratio of volumes of the neighbouring cells – in the case where the weights of the random generator are not uniformly bounded. As it turns out, it is useful to study the behaviour of the distance to the furthest neighbour of a typical point of the point process. In this contribution we will present some properties of this characteristic, which were derived using the concept of tempered configurations.

Liquid-gas phase transition for Gibbs point processes with saturated interaction

Christopher Renaud Chan
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Gibbs point processes are natural objects to study systems of particles in interaction. In finite volume, the unnormalised density of the Gibbs measure with respect to a Poisson point process with activity z is given by the Boltzmann factor $e^{-\beta H}$, where β is the inverse temperature and H is the Hamiltonian that encodes the interaction between particles. The infinite volume Gibbs point processes are defined as solutions to the Dobrushin-Lanford-Ruelle equations, which describe the equilibrium of the system. A liquid-gas phase transition occurs when we do not have uniqueness of the infinite volume Gibbs point process and that they have different densities. We explore the occurrence of such phenomenon in the context of saturated interactions. These interactions represent a class of models where the energy cost of adding a point in areas of high particle density is constant. The Quermass interaction exhibit such saturation property. We will present another interesting example: the diluted pairwise interaction. Under some assumptions, we prove the existence of liquid-gas phase transition for saturated interactions using an adaptation of Pirogov-Sinaï-Zahradník theory in the continuous setting.

A min-max random game on a graph that is not a tree

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In a classical game two players, Alice and Bob, take turns to play n moves each. Alice starts. For each move each player has two options, 1 and 2. The outcome is determined by the exact sequences of moves played by each player. Prior to the game, a winner is assigned to each of the 2^{2n} possible outcomes in an i.i.d. fashion, where p is the probability that Bob is the winner for a given outcome. Then it is known that there exists a value $0 < p_c < 1$ such that the probability that Bob has a winning strategy for large n tends to one if $p > p_c$ and to zero if $p < p_c$. We study a modification of this game for which the outcome is determined by the exact sequence of moves played by Alice as before, but in the case of Bob all that matters is how often he has played move 1. We show that also in this case, there exists a sharp threshold p_c' that determines which player has with large probability a winning strategy in the limit as n tends to infinity. Joint work with Anja Sturm and Natalia Cardona-Tobón (Göttingen).
